

# Muonium Spectroscopy

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The electromagnetic interactions of electrons and muons can be described to very high accuracy within the framework of standard theory, in particular within the hydrogen-like muonium atom. Therefore precision measurements allow to test basic interactions in physics and to search for yet unknown forces. Accurate values for fundamental constants can be obtained. Results from experiments on the ground state hyperfine structure and the 1s-2s intervals in muonium are described together with their relations to a new measurement of the muon magnetic anomaly.

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## 1. Introduction

To present knowledge leptons have dimensions of less than  $10^{-18}m$  and may therefore be regarded as point-like objects. The muonium atom ( $M = \mu^+e^-$ ) is the hydrogen-like bound state of leptons from two different particle generations, an antimuon( $\mu^+$ ) and an electron( $e^-$ ) [1,2]. The dominant interaction within the M atom is electromagnetic and level energies can be calculated in bound state Quantum Electrodynamics (QED) to sufficiently high accuracy for modern high precision spectroscopic experiments. There are also contributions from weak interactions arising from  $Z^0$ -boson exchange and from strong interactions due to vacuum polarization loops containing hadrons. They both can be obtained to the required level of precision using standard theory. In contrast to natural atoms and ions as well as artificial atomic systems, which contain hadrons, M has the advantage that there are no complications arising from the finite size and the internal structure of any of its constituents. Precision experiments in M can therefore provide sensitive tests of the standard theory and searches for new and yet unknown forces in nature. Parameters of speculative theories, which try to expand the standard model in order to gain deeper insight into some of its not

well understood features, can be restricted. In addition, fundamental constants like the muon mass  $m_\mu$ , its magnetic moment  $\mu_\mu$  and anomaly  $a_\mu$  and the fine structure constant  $\alpha$  can be obtained.

All high precision experiments in M up to date atom have involved the 1s ground state (see Fig.1), in which the atoms can be produced in sufficient quantities [2]. The most efficient mechanism is  $e^-$  capture after stopping  $\mu^+$  in a suitable noble gas, where yields of 80(10)% were achieved for Kr gas [1]. This technique was used in the most recent precision measurements of the atom's ground state hyperfine structure splitting  $\Delta\nu_{HFS}$  and  $\mu_\mu$  at the Los Alamos Meson Physics Facility (LAMPF) in Los Alamos, USA [3]. Muonium at thermal velocities in vacuum can be obtained by stopping  $\mu^+$  close to the surface of a  $\text{SiO}_2$  powder target, where the atoms are formed through  $e^-$  capture and some of which diffuse through the target surface into the surrounding vacuum. This process has an efficiency of a few % and was an essential prerequisite for Doppler-free two-photon laser spectroscopy of the  $1^2\text{S}_{1/2}$ - $2^2\text{S}_{1/2}$  interval  $\Delta\nu_{1s2s}$  at the Rutherford Appleton Laboratory (RAL) in Chilton, United Kingdom [4], which yields an accurate value for  $m_\mu$ . Electromagnetic transitions in excited states, particularly the  $2^2\text{S}_{1/2}$ - $2^2\text{P}_{1/2}$  classical Lamb shift and  $2^2\text{S}_{1/2}$ - $2^2\text{P}_{3/2}$  fine structure splitting could be induced by microwave spectroscopy. However, because only moderate numbers of atoms in the metastable 2s state can be produced with a beam foil technique, the experimental accuracy is now the 1.5 % level [5,6], which represents not yet a severe test of theory.

## 2. Ground State Hyperfine Structure

The most recent experiment at LAMPF used a Kr gas target inside of a microwave cavity at typically atmospheric density and in a homogeneous magnetic field of 1.7 Tesla. Microwave transitions between the two energetically highest respectively two lowest Zeeman sublevels of the  $n=1$  state at the frequencies  $\nu_{12}$  and  $\nu_{34}$  (Fig.1) involve a muon spin flip. They were detected through a change in the spatial distribution of  $e^+$  from  $\mu^+$  decays, since due to parity violation in the  $\mu^+$  decay the  $e^+$  are preferentially emitted in the  $\mu^+$  spin direction. As a consequence of the Breit-Rabi equation, which describes the behaviour of the levels in a magnetic field, the sum of these frequencies equals at any field value the splitting in zero field  $\Delta\nu_{HFS}$  and their difference yields in a known field  $\mu_\mu$ . The experiment utilized the technique of "old muonium", which al-

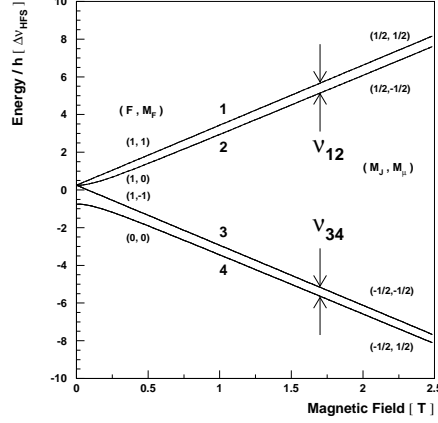


Figure 1. Left: Muonium n=1 and n=2 states. All indicated transitions could be induced to date. Right: Ground state Zeeman levels in an external magnetic field.

lowed to reduce the linewidth of the signals below half of the "natural" linewidth  $\delta\nu_{nat} = (\pi \cdot \tau_\mu)^{-1} = 145 \text{ kHz}$ , where  $\tau_\mu$  is the muon lifetime of  $2.2 \mu$  (Fig.2). For this purpose an essentially continuous muon beam was chopped by an electrostatic kicking device into  $4 \mu\text{s}$  long pulses with  $14 \mu\text{s}$  separation. Only atoms which were interacting coherently with the microwave field for periods longer than several muon lifetimes were detected [7].

The results are mainly statistics limited and improve the knowledge of both  $\Delta\nu_{HFS}$  and  $\mu_\mu$  by a factor of three [3] over previous measurements [8]. The zero field splitting is determined to  $\Delta\nu_{HFS} = \nu_{12} + \nu_{34} = 4\,463\,302\,765(53) \text{ Hz}$  (12 ppb) which agrees well with the theoretical prediction of  $\Delta\nu_{theory} = 4\,463\,302\,563(520)(34)(\leq 100) \text{ Hz}$  (120 ppb) [10]. Here the first quoted uncertainty is due to the accuracy to which the muon-electron mass ratio  $m_\mu/m_e$  is known, the second error is from the knowledge of  $\alpha$  as obtained in electron g-2 measurements, and the third value corresponds to estimates of uncalculated higher order terms. The strong interaction contributes 250 Hz and a parity conserving weak interaction amounts to -65 Hz. Among the possible exotic interactions which could contribute to  $\Delta\nu_{HFS}$  is the conversion of muonium to antimuonium, which is in the lepton sector an analogous process to the well known  $K_0-\bar{K}_0$  oscillations in the quark sector. From a recent direct search at the Paul Scherrer Institute (PSI) in Villigen, Switzerland, which itself could significantly restrict several

speculative models, an upper limit of 9 Hz can be concluded for an expected line splitting [9,2]. Recently generic extensions of the standard model in which both Lorentz invariance and CPT invariance are not assumed have attracted widespread attention in physics [11]. Such models suggest diurnal variations of the ratio  $(\Delta\nu_{12} - \Delta\nu_{34})/(\Delta\nu_{12} + \Delta\nu_{34})$  [12] which are being searched for [13].

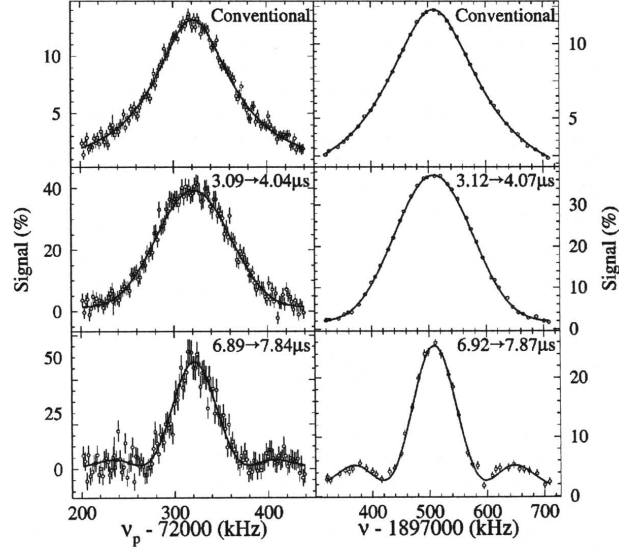


Figure 2. Samples of conventional and ‘old’ M resonances at frequency  $\nu_{12}$ . The narrow ‘old’ lines are also higher. The lines in right column were recorded using a sweep of the magnetic field, which was measured in units of the proton NMR frequency  $\nu_P$ . The lines to the left were obtained using microwave frequency scans.

The magnetic moment results from the measurements as  $\mu_\mu/\mu_p = 3.183\,345\,24(37)$  (120 ppb) which translates into  $m_\mu/m_e = 206.768\,277(24)$  (120 ppb). The hyperfine splitting is proportional to  $\alpha^2 R_\infty$ , with the very precisely known Rydberg constant  $R_\infty$ . Comparing experiment and theory yields  $\alpha_2^{-1} = 137.035\,996\,3(80)$  (58 ppb) [3]. If  $R_\infty$  is decomposed into even more fundamental constants, one finds  $\Delta\nu_{HFS}$  to be proportional to  $\alpha^4 m_e/\hbar$ . Using the value  $\hbar/m_e$  as determined in measurements of the neutron de Broglie wavelength [14] gives  $\alpha_4^{-1} = 137.036\,004\,7(48)$  (35 ppb). In the near future a small improvement in  $\alpha_4^{-1}$  can be expected from ongoing determinations of  $\hbar/m_e$  in measurements of the photon recoil in Cs atom spectroscopy and a Cs atomic mass measurement. The present limitation for accuracy of  $\alpha_4^{-1}$  arises mainly from the muon mass uncertainty. Therefore any better determination of the muon mass, e.g. through a precise

measurement of the reduced mass shift in  $\Delta\nu_{1s2s}$ , will result in an improvement of  $\alpha_4^{-1}$ . At present the good agreement within two standard deviations between the fine structure constant determined from M hyperfine structure and the one from the electron magnetic anomaly is generally considered the best test of internal consistency of QED, as one case involves bound state QED and the other one QED of free particles.

### 3. 1s-2s Energy Interval

Doppler-free excitation of the 1s-2s transition has been achieved in the past at KEK in Tsukuba, Japan, [15] and at RAL [16]. The accuracy of the latter measurement was limited by ac Stark effect and a frequency chirp caused by rapid changes of the index of refraction in the dye solutions of the amplifier stages in the employed high power laser system. A new measurement has been performed very recently at the worlds brightest pulsed surface muon source at RAL [4].

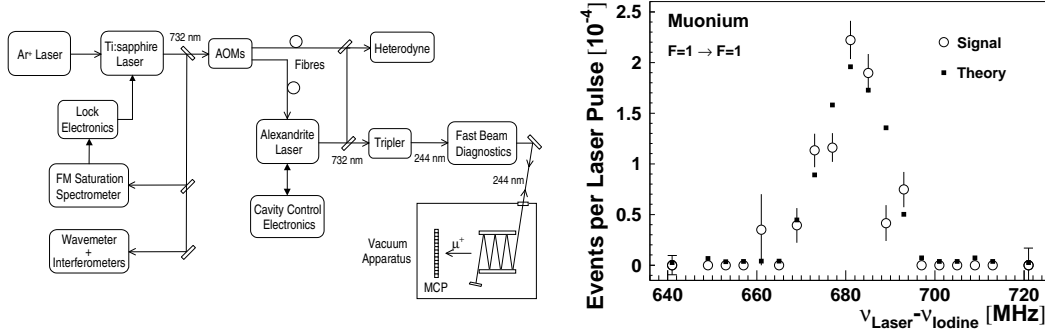


Figure 3. Left: Pulsed laser system in the M 1s-2s experiment. Right: Muonium 1s-2s signal. The frequency corresponds to the offset of the Ti:sapphire laser from the iodine reference line. The open circles are the observed signal, the solid squares represent the theoretical expectation based on measured laser beam parameters and a line shape model [17].

The  $1^2S_{1/2}(F=1) \rightarrow 2^2S_{1/2}(F=1)$  transition was induced when thermal muonium atoms interacted with the light field of two counter-propagating laser beams of wavelength 244 nm. The two-photon excitation was detected by photoionization of the 2s state in the same light field. The muons released thereby were identified and counted. Their number as a function of laser frequency represents the experimental signal (Fig.3). The necessary high power UV laser light was

generated by frequency tripling the output of an alexandrite ring laser amplifier in crystals of LBO and BBO. The alexandrite laser was seeded with light from a continuous wave Ar ion laser pumped Ti:sapphire laser at 732 nm. Fluctuations of the optical phase during the laser pulse (chirping) were compensated with two electro-optic devices in the resonator of the ring amplifier to give a swing of the laser lights frequency chirping of less than about 5 MHz. The fundamental optical frequency was calibrated by frequency modulation saturation spectroscopy of the  $a_{15}$  hyperfine component of the 5-13 R(26) line in thermally excited  $^{127}\text{I}_2$  vapour which lies about 700 MHz lower than  $1/6$  of the M transition frequency. It has been calibrated to 0.4 MHz [18]. The cw light was frequency up-shifted by passing through two acousto-optic modulators (AOM's).

The experiment yields  $\Delta\nu_{1s2s}(\text{expt.}) = 2\,455\,528\,941.0(9.8)$  MHz in good agreement with a theoretical value of  $\Delta\nu_{1s2s}(\text{theory}) = 2\,455\,528\,935.4(1.4)$  MHz [19]. From these values the muon-electron mass ratio is found to be  $m_{\mu^+}/m_{e^-} = 206.768\,38(17)$ . Alternatively, using  $m_{\mu^+}/m_{e^-}$  extracted from the M hyperfine structure experiment a comparison of  $\Delta\nu_{1s2s}(\text{expt.})$  and  $\Delta\nu_{1s2s}(\text{theory})$  yields the  $\mu^+-e^-$  charge ratio as  $Z = q_{\mu^+}/q_{e^-} = -1 - 1.1(2.1) \cdot 10^{-9}$ . This is the best verification of charge equality in the first two generations of particles. The existence of one single universal quantized unit of charge is solely an experimental fact for which no associated underlying symmetry has yet been revealed. Gauge invariance assures charge quantization only within one generation of particles.

#### 4. Muon Magnetic Anomaly

The muon magnetic anomaly  $a_\mu$  is given, like in case of the electron, mostly by photon and by electron-positron fields. However, the effects of heavier particles is enhanced by the square of the mass ratio  $m_\mu/m_e \approx 4 \cdot 10^4$ . The contributions of the strong interaction, which can be determined from a dispersion relation with the input from experimental data on  $e^+-e^-$  annihilation into hadrons and hadronic  $\tau$ -decays, amounts to 58 ppm. The weak interaction adds 1.3 ppm. At present standard theory yields  $a_\mu$  to 0.66 ppm. Contributions from physics beyond the standard model may be as large as a few ppm. Such could arise from, e.g., supersymmetry, compositeness of fundamental fermions and bosons, CPT violation and many others.

A new determination of  $a_\mu$  [20] is presently carried out in a superfermic magnetic storage Ring [21] at the Brookhaven National Laboratory (BNL) in

Upton, USA. It is a g-2 experiment in which the difference of the spin precession and the cyclotron frequencies is measured. In a first startup run, approximately the same level of accuracy for  $\mu^+$  could be reached as the final result for this particle in a preceding experiment at CERN [22]. Several technical improvements were installed since, the most significant of which is a magnetic kicker, which allows to inject muons directly into the storage ring. This enhances the number of stored particles by almost two orders of magnitude compared to the early stages of the experiment when the stored muons were born in the decays of injected pions. Data have been taken which are expected to yield  $a_\mu$  to 1 ppm. The data analyzed so far have give the value with 5 ppm uncertainty. The value agrees with the prediction of standard theory. The experiment aims for a final precision of 0.35 ppm. To be able to reach this goal, it is essential to have  $\mu_\mu$  to the 0.1 ppm level from muonium spectroscopy, since this quantity is important in the extraction of the experimental result.

The experiment is planed for both  $\mu^+$  and  $\mu^-$  as a test of CPT invariance. This is of particular interest in view of the suggestion by Bluhm et al. [11] and Dehmelt et al. [23] to compare tests of CPT invariance in different systems on a common basis, i.e. the energies of the involved states. For measurements of magnetic anomalies this means that the energies of particles with spin down in an external field need to be compared to the energies of antiparticles with spin up. The nature of g-2 experiments is such that they provide a figure of merit  $r = |a^- - a^+| \cdot \frac{\hbar\omega_c}{m \cdot c^2}$  for a CPT test, where  $a^-$  and  $a^+$  are the positive and negative particles magnetic anomalies,  $\omega_c$  is the cyclotron frequency used in the measurement and  $m$  is the particle mass. For the past electron and positron measurements one has  $r_e = 1.2 \cdot 10^{-21}$  [23] which is a much tighter test than in the case of the neutral kaon system, were the mass differences between  $K^0$  and  $\overline{K}^0$  yield  $r_K = 1 \cdot 10^{-18}$ . An even more stringent CPT test arises from the past muon magnetic anomaly measurements were  $r_\mu = 3.5 \cdot 10^{-24}$ , which may therefore already be viewed as the presently best known CPT test based on system energies. With improvement expected in the BNL g-2 experiment one can look forward to a 20 times more precise test of this fundamental symmetry.

## 5. Future possibilities

All precision M experiments are now limited by statistics. Therefore significant improvements can be expected from either more efficient M formation,

Table 1

Muon fluxes of some existing and future facilities, Rutherford Appleton Laboratory (RAL), Japanese Hadron Facility (JHF), a new Neutron Spallation Source (NSS), Muon collider (MC).

	RAL( $\mu^+$ )	PSI( $\mu^+$ )	PSI( $\mu^-$ )	JHF( $\mu^+$ ) <sup>†</sup>	NSS( $\mu^+$ )	MC ( $\mu^+$ , $\mu^-$ )
Intensity ( $\mu/s$ )	$3 \times 10^6$	$3 \times 10^8$	$1 \times 10^8$	$4.5 \times 10^{11}$	$4.5 \times 10^7$	$7.5 \times 10^{13}$
Momentum bite $\Delta p/p[\%]$	10	10	10	10	10	5-10
Spot size (cm $\times$ cm)	1.2 $\times$ 2.0	3.3 $\times$ 2.0	3.3 $\times$ 2.0	1.5 $\times$ 2.0	1.5 $\times$ 2.0	few $\times$ few
Pulse structure	82 ns	50 MHz	50 MHz	300 ns	300 ns	50 ps
	50 Hz	contin.	contin.	50 Hz	50 Hz	15 Hz

which might in principle be possible to a small extent in the case of thermal M in vacuum. The best solution, however, would be muon sources of higher intensities. Such may become available in the intermediate future the Japanese Hadron Facility (JHF), or the Oak Ridge (or a possible European) Spallation Neutron Source (NSS) Also the discussed Oak Ridge neutron spallation source. The most promising facility is, however, a muon collider [24]; its front end will provide muon rates 5-6 orders of magnitude higher than present beams (Table 4).

At such facilities there is in addition to more precise measurements in M a variety of experiments on artificial atoms and ions like muonic hydrogen and muonic helium which will allow to extract important parameters describing the hadronic particles within these systems or fundamental interactions, which could in no physical experiment thus far be accessed with sufficient precision for atomic, nuclear and particle theory [25,26]. It should be noted that new experimental approaches [9,27] would also become feasible which might beneficially take advantage of, e.g., the time evolution of the atomic systems.

## 6. Conclusions

Although the nature of the muon - the reason for its existence - still remains a mystery, both the theoretical and experimental work in fundamental muon physics, have contributed to an improved understanding of basic particle interactions and symmetries in physics. Particularly muonium spectroscopy has verified the nature of the muon as a point-like heavy lepton which differs only in its mass related parameters from the others. This fact is fundamentally assumed in every precision calculation within standard theory. In addition, the measure-



ments provide accurate values of fundamental constants.

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